How do we evaluate limit?

Prove

$$\lim_{x \to 5} 4x+2 = 22$$

Given $\varepsilon > 0$

Define that $\delta = \frac{\varepsilon}{4}$

Assume $|x-5| < \delta$

$|x-5| < \delta$

$4|x-5| < 4\delta$

$|4x-20| < 4\delta$

$|(4x+2)-22| < 4\delta = 4\left(\frac{\varepsilon}{4}\right) = \varepsilon$

$\therefore \lim_{x \to 5} 4x+2 = 22$

S.W.

$f(x) = 4|x+2$

$|x-5| < \delta$

$|4|x-20| < \varepsilon$

$4|x-5| < \varepsilon$

$|x-5| < \frac{\varepsilon}{4}$

if $\delta = \frac{\varepsilon}{4}$

$\lim_{x \to 3} x^2 - 4 = 5$

To prove

Given: $\varepsilon > 0$

Define: $\delta = \min\left\{1, \frac{\varepsilon}{7}\right\}$

Assume $|x-3| < \delta$

$|x+3||x-3| < 7|x-3| < 7\delta$

$|x+3||x-3| < 7\delta$

$|x^2 - 9| < 7\delta$

$|(x^2-4)-5| < 7\delta = 7\left(\frac{\varepsilon}{7}\right) = \varepsilon$

$\therefore \lim_{x \to 3} x^2 - 4 = 5$

$|x-3| < \delta$

$|(x^2-4)-5| < \varepsilon$

$|x^2 - 9| < \varepsilon$

$x \to 3$

$|x+3|\,|x-3| < \varepsilon$

$x+3$

$|x-3| < \frac{\varepsilon}{|x+3|} < \frac{\varepsilon}{5}$

$\frac{\varepsilon}{|x+3|} < \frac{\varepsilon}{4}$

$\lim_{x \to 3} x^2 + 2x = 15$

To prove

Given: $\varepsilon > 0$

Define $\delta = \min\left\{1, \frac{\varepsilon}{9}\right\}$

Assume $|x-3| < \delta$

$|x-3| < \delta$

$|x^2+2x-15| < \varepsilon$

$|x-3|\underbrace{|x+5|}_{\downarrow 9} < \varepsilon$

$\delta = \frac{\varepsilon}{9}$

$|x-3| < \frac{\varepsilon}{(x+5) \to 9}$

$|x+5||x-3| < 9|x-3| < 9\delta = 9\left(\frac{\varepsilon}{9}\right) = \varepsilon$

$|x^2+2x-15| < \varepsilon$

$\therefore \lim_{x \to 3} x^2 + 2x = 15$